Work Energy Packet 2

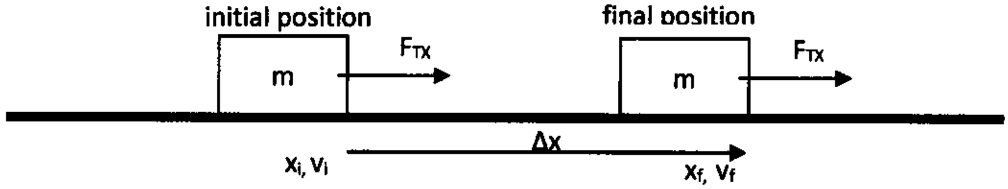
Work and Kinetic Energy

Work and energy are closely related. When you do work on a mass, you change the energy of the mass. But what exactly is "energy"? Energy turns out to be a bit difficult to define simply and clearly. A traditional definition of energy is "the ability to do work." We'll start with that. That means, if something has energy, it can exert a force on something else over a distance, ie move it.

Energy comes in different forms. If something is moving, it has "kinetic" energy, or energy of motion. Other forms of energy include potential energy (a mass has more energy if it is lifted off the ground than if it is at rest on the ground), thermal energy (hot = more energy) and others. This packet will limit the discussion to kinetic energy.

Here's the big idea. It turns out that the total amount of energy in the universe is a fixed number. An object can transform kinetic to potential energy, or transfer its energy to another object, but energy cannot be created or destroyed. If you sit back and think about it, that's amazing. (Einstein broadened this even further by showing the equivalence of mass and energy: $E = mc^2$, but that will have to wait for another course). This packet is about the relationship between Work and Energy. The idea that the total energy is a fixed number is the next packet.

Let's go back to the opening statement "when you do work on a mass, you change the energy of the mass" and explore it a bit. Consider straight line motion in the x direction as shown in the following diagram. F_{TX} represents all the "x" forces already combined into the single constant F_{TX}. A complete free body diagram of the mass m at the initial and final positions would include both weight (F_g) and the normal force, but neither F_g nor F_N has a component in the horizontal direction (they're all in the vertical direction), so neither contributes to F_{TX}. So for clarity, they are left off.



Recall, Newton's 2nd Law: (in the direction of motion)

Equation 1 $$F_{TX} = m \cdot a_x$$

Since the force is constant, so is the acceleration (due to Newton's 2nd Law), meaning we can use the kinematics equations we've been using all year long (recall, they only work for constant acceleration).

$$v_{xf}^2 = v_{xi}^2 + 2a_x\Delta x$$

Solving this for a_x yields:

$$a_x = \frac{v_{xf}^2 - v_{xi}^2}{2\Delta x}$$

Plugging this a_x into equation 1 yields:

$$F_{TX} = m(a_x)$$

$$F_{Tx} = m\left(\frac{v_{xf}^2 - v_{xi}^2}{2\Delta x}\right)$$

$$F_{Tx}\Delta x = \frac{1}{2}m\left(v_{xf}^2 - v_{xi}^2\right)$$

Equation 2 $$W_T = \frac{1}{2}mv_{xf}^2 - \frac{1}{2}mv_{xi}^2$$

The *W* term on the left is our old friend "Work" and the term *½ mv²* term on the right is our new friend, the "kinetic energy" (KE) of mass m, or the energy associated with its velocity. Equation 2 can thus be restated as:

Equation 3 $$W = KE_f - KE_i = \Delta KE$$

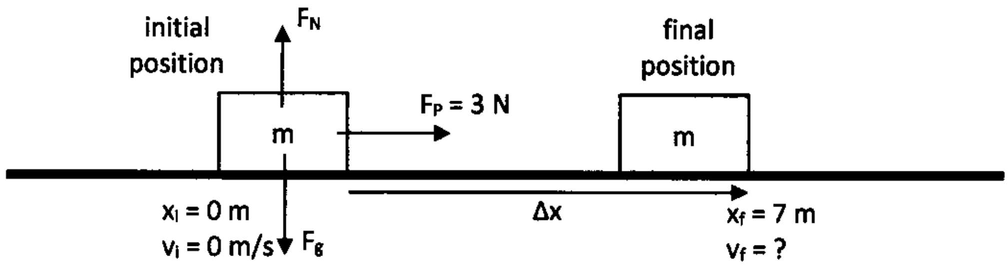
Kinetic Energy = ½mv²

Equation 3 tells us that when work is done on a mass, the amount of work equals the change in kinetic energy of the mass. This is called the "Work – Kinetic Energy Theorem". Though we derived it for horizontal motion, it applies for any motion

Work – Kinetic Energy Theorem $W_T = \Delta KE = KE_f - KE_i$

Example 1.

A 1 kg mass, initially at rest, is pulled to the right with a horizontal force of 3 N for 7 m over a level frictionless surface. How fast is the mass going at the end of those 7 m?



Solution 1: From the Work – Kinetic Energy Theorem: $W_T = \Delta KE = KE_f - KE_i$

$$W_T = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$F_{Tx}\Delta x = \frac{1}{2}mv_{xf}^2 - \frac{1}{2}mv_{xi}^2$$

Since F_N and F_g do not have any components in the horizontal direction and F_P is completely horizontal, $F_{TX} = F_P = 3$ N. Now, put in all the numbers and solve for V_{xf}.

$$(3\text{ N})(7\text{ m}) = ½\,(1\text{ Kg})(v_{xf})^2 + ½\,(1\text{ Kg})(0\text{ m/s})^2$$

$$21 = ½\,(v_{xf})^2 + 0$$

$$v_{xf}^2 = \sqrt{42}$$

$$v_{xf} = 6.48\text{ m/s}$$

That's how to use the Work-Kinetic Energy Theorem to relate work to kinetic energy, and thus force to velocity.

Solution 2: You could also solve this from Newton's Laws and the kinematics equations:

$$F_{TX} = ma_x$$

$$3\text{ N} = 1\text{ Kg}\,(a_x)$$

$$a_x = 3\text{ m/s}^2$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x\Delta x$$

$$v_{xf}^2 = 0 + 2(3\frac{\text{m}}{\text{s}^2})(7\text{m})$$

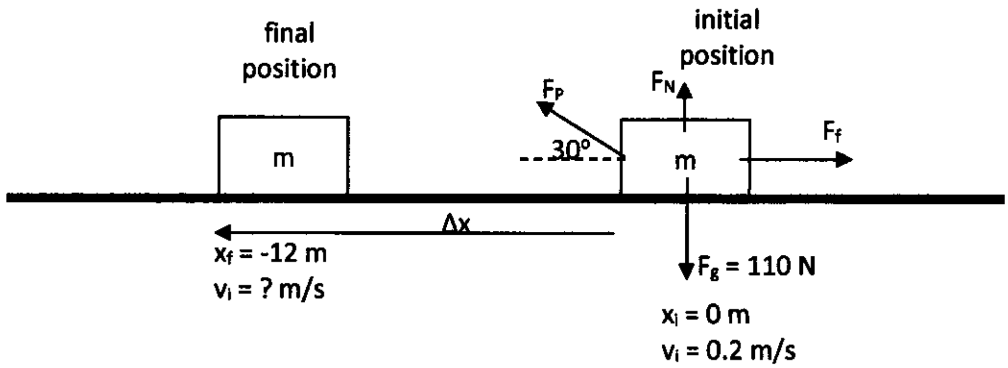
$$v_{xf} = 6.48\text{ m/s}$$

In general, the work – energy approach requires fewer steps to reach a solution than Newton's Law / kinematics.

Here's a more complicated example on the force side. But once you get the total force, the work energy part is the same.

Example 2.

A 11 kg sled, initially travelling at 0.2 m/s to the left, is pulled by a 15 N force to the left at an angle of 30° above horizontal for a straight line distance of 12 m. The coefficient of friction between the sled and the ice is 0.08. How fast is the mass going at the end of those 12 m?



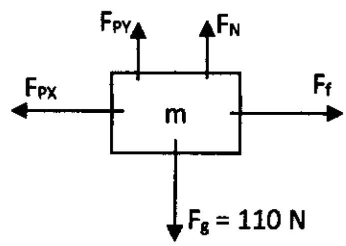
Solution: From the Work – Kinetic Energy Theorem: $W_T = \Delta KE = KE_f - KE_i$

$$W_T = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$F_{Tx}\Delta x = \frac{1}{2}mv_{xf}^2 - \frac{1}{2}mv_{xi}^2$$

So, to get F_{TX}, first do the trig and redraw FBD:

$F_{Px} = 15\cos(30°) = 12.99$ N
$F_{Py} = 15\sin(30°) = 7.5$ N



Then, from the redrawn FBD:

$F_{TX} = F_f + F_{Px}$ → need F_f → $F_f = \mu F_N$ → need F_N → $F_{TY} = 0$, $F_{TY} = F_N + F_{Py} + F_g$

$F_N + 7.5 - 110 = 0$
$F_N = 102.5$ N

$F_f = (0.08)(102.5\text{ N})$
$F_f = 8.2$ N

$F_{TX} = 8.2 - 12.99$
$F_{TX} = -4.79$ N,
or 4.79 N left

That's F_{TX}. (-4.79 N)
$\Delta x = (x_f - x_i) = (-12\text{ m} - 0\text{ m}) = -12$ m
Plugging these into the Work – Kinetic Energy theorem yields:

$$F_{Tx}\Delta x = \frac{1}{2}mv_{xf}^2 - \frac{1}{2}mv_{xi}^2$$

$(-4.79\text{ N})(-12\text{ m}) = \frac{1}{2}(11\text{ kg})(v_{xf})^2 - \frac{1}{2}(11\text{ kg})(0\text{ m/s})^2$ (→ 0)
$57.48 = 5.5(v_{xf})^2$
$v_{xf} = +/- 3.23$ m/s

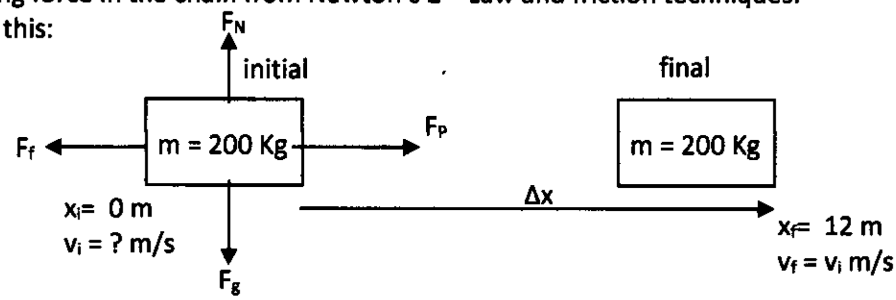
Choose negative since object is moving left

Example 3

A farmer pulls a 200 Kg pallet of fertilizer across the level barn floor at a constant velocity with his tractor for 12 m. The pallet is attached to the tractor with a chain that is horizontal. The coefficient of friction between the pallet and the floor is 0.24.

a) How much work does the chain do on the pallet?
b) How much work does the floor do on the pallet (through friction)?
c) What is the total work done on the pallet?

Solution a) First, find the pulling force in the chain from Newton's 2[nd] Law and friction techniques.
The FBD looks like this:



Since the pallet is moving at a constant velocity in the x direction, from the 1[st] Law: $F_{TX} = 0$ and $F_{TY} = 0$, making

$F_{TX} = 0$
$F_{TX} = F_f + F_P$. ⟶ need F_f ⟶ $F_f = \mu F_N$ ⟶ need F_N ⟶ $F_{TY} = 0$, $F_{TY} = F_N + F_g$, $F_N - 2000 = 0$, $F_N = 2000$ N
⟵ $F_f = (0.24)(2000)$, $F_f = 480$ N
⟵ $0 = -480 + F_P$
$F_P = 480$ N,

Recall, work done by a force is the component of the force in the direction displacement times that displacement. Since the chain force F_P is horizontal and the displacement is also horizontal, the entire F_P is on the direction of displacement. So work done on the pallet by the chain is:

$$W_{Fp} = F_p \cdot \Delta x$$
$$W = (480\ N)(12\ m)$$
$$W = 5{,}760\ J$$

b) How much work does the floor do on the pallet (through friction)?

$$W_{Ff} = F_f \cdot \Delta x$$
$$W = (-480\ N)(12\ m)$$
$$W = -5{,}760\ J$$

c) What is the total work done on the pallet?

The total work is the sum of the work done by each individual force, meaning:

$$W_T = W_{FN} + W_{Fg} + W_{FP} + W_{Ff}$$

Since weight (F_g) and normal force (F_N) do not have a component in the direction of displacement, their contribution to work in the horizontal direction is zero. So:

$$W_T = 0 + 0 + 480\ J - 480\ J$$
$$W_T = 0$$

You can reach the same conclusion directly from the Work – Kinetic Energy theorem without having to solve for all the individual forces as follows:

From the Work – Kinetic Energy Theorem: $W_T = KE_f - KE_i = \Delta KE$

$$W_T = \frac{1}{2}mv_{xf}^2 - \frac{1}{2}mv_{xi}^2$$

Since the pallet is moving at constant velocity, $v_{xf} = v_{xi}$. Let's just call it v_x since it never changes.

Then the work energy theorem gives us:

$$W_T = \frac{1}{2}mv_{xf}^2 - \frac{1}{2}mv_{xi}^2$$

$$W_T = \frac{1}{2}mv_x^2 - \frac{1}{2}mv_x^2$$

$$W_T = 0$$

Summary:

Total work done on an object equals the change in kinetic energy of the object. If no total work is done on an object, there is no change in kinetic energy, meaning no change in speed. The converse holds true as well: If there is no change in kinetic energy of an object (meaning no change of speed), there is no total work done on the object.

Like Work, Kinetic energy is a scalar, not a vector.

If the total work on an object is positive, the object experiences an increase in kinetic energy. Since kinetic energy is ½ mv^2, an increase in kinetic energy means the object speeds up.

If the total work on an object is negative, the object experiences an decrease in kinetic energy. A decrease in kinetic energy means the object slows down.

General problem solving guidelines with work energy:

1. Determine problem type. If you have forces, displacements (distances), velocities, and work or energies (ie something in Joules), the problem is a good candidate for a work energy strategy.
2. Draw a FBD
4. Use the Work – Kinetic Energy Theorem to set the total work done equal to the change in kinetic energy:

   $W_T = KE_f - KE_i = \Delta KE$

   This relates total force in direction of motion and displacement to mass and velocity
5. Solve for the appropriate variable.

1. 32.4 J
2. a. 140,000 J b. 560,000 J 420 kJ 420 kJ
3. 1.73 m/s
4. a. No. It takes 28.57 m to stop b. Yup. It would have taken you 114.28 m to stop c. 4 e. 63 m
5. a. 288 N b. 460.8 J d. 0.98 m/s
6. 1.54 m/s
7. - 11.83 m/s
8. 31.62 m/s down

Energy Pack 2 Questions

Practice Questions (answers on page 5)

Use the Work Kinetic Energy Theorem and <u>NOT</u> kinematics equations everywhere in this packet

1. What is the kinetic energy of a 0.2 kg baseball travelling at a velocity of 18 m/s?

2. What is the kinetic energy of a 700 kg car traveling in a straight line at:

   a. 20 m/s (which is about 45 mph)

   b. 40 m/s (which is about 90 mph)

   c. What is the (arithmetic) difference in kinetic energy between a) and b)?

d. How much work must be done on the car in part a) to change its energy to that of part b)?

3. A 4 kg block, initially at rest, is pulled along a frictionless surface to the right in a straight line for 3 m with horizontal force to the right of 2 N. How fast is the block going at the end of the 3 m?

4. You are driving along a level road in a straight line in your 800 kg car at a speed of 20 m/s. The coefficient of friction between your tires and the pavement is 0.7. A tree falls on the road 40 m directly in front of you, forcing you to stomp on your brakes.

   a. Do you hit the tree?

   b. Same scenario, only you are going 40 m/s when you hit your brakes. Do you hit the tree?

   c. From part a to part b, your speed increased by a factor of 2. By what factor did your stopping distance increase?

d. What's the relationship between the stopping distance and the speed at which you apply the brakes?

e. If your car had a mass and coefficient of friction that led to a stopping distance of 7 m when braking from 10 m/s, what would be the stopping distance when braking from 30 m/s?

5. You are pushing an 80 kg entertainment center across a rug. The coefficient of friction between the entertainment center and the rug is 0.36 (assume static and kinetic coefficients of friction are the same)

    a. How hard must you push the entertainment center to get it moving?

b. If you push at the minimum force required to keep the entertainment center moving at constant velocity for 1.6 m. how much work do you do on the entertainment center?

c. What is the total work done on the entertainment center?

d. Same scenario, only the coefficient of static friction is 0.36 and the coefficient of kinetic friction is 0.33. If you push with the force of part a) to get the entertainment center moving, how fast is it going at the end of 1.6 m?

6. Strange things are afoot at the Circle K. Bill and Ted pull a 50 Kg phone booth, initially at rest, across the level parking lot in a straight line for 4 m with a cable angled at 40° above horizontal. They pull on the cable with a force of 300 N. The coefficient of friction between the phone booth and the parking lot is 0.70. How fast is the phone booth going at the end of the 4 m?

7. How fast is a 0.43 kg ball going just before it hits the ground if it is dropped from a height of 7 m?

8. You drop a ball from a height of 50 m. How fast is it going just before it hits the ground? (As everywhere in this packet, no kinematics!)

9. A 4 kg block slides down along a frictionless ramp which is angled 20° above horizontal. If the block starts from rest, how fast is it going after sliding 5 m?

10. A 4 kg block is pulled 4.7 m up the surface of a ramp which is angled 30° above horizontal by a 35 N force which acts parallel to the surface of the ramp. The coefficient of friction between the block and the ramp is 0.1. If the block is traveling at 5.3 m/s at the end of the 4.7 m pull, how fast was it going at the beginning?

11. You failed to recollect that the coefficient of friction goes down when the pavement is wet and took a corner too fast. The lower force of friction was unable to provide the centripetal force you demanded at that speed, which resulted in your 1,000 kg car sliding off the road into a ditch. Confused as to how to extricate your car from the ditch, you call your Dad, who calls a tow truck. The tow truck pulls your car out of the ditch, 5 m up the surface of a hill which is angled 20° above horizontal with a chain which is parallel to the surface of the hill. The chain exerts a constant force on the car. The coefficient of friction between the car and the hill is 0.9. If the car starts from rest and is traveling at 1.3 m/s at the end of the 5 m pull, with what force did the chain pull on the car?

12. You drop a ball from a height of 50 m. How fast is it going just before it hits the ground? (As everywhere in this packet, no kinematics!)

13. A block slides 7 m down the surface of a frictionless ramp which is angled 20° above horizontal. If the block starts from rest, how fast is it going after the 7 m?